Nuclear structure and the 'proton radius puzzle'

Nir Nevo Dinur[1,2]
Chen Ji[2,3], **Javier Hernandez**[2,5], **Sonia Bacca**[2,4], **Nir Barnea**[1]

[1]The Hebrew University of Jerusalem, Israel
[2]TRIUMF, Vancouver, BC, Canada
[3]ECT* and INFN-TIFPA, Trento, Italy
[4]University of Manitoba, Winnipeg, Canada
[5]University of British Columbia, BC, Canada

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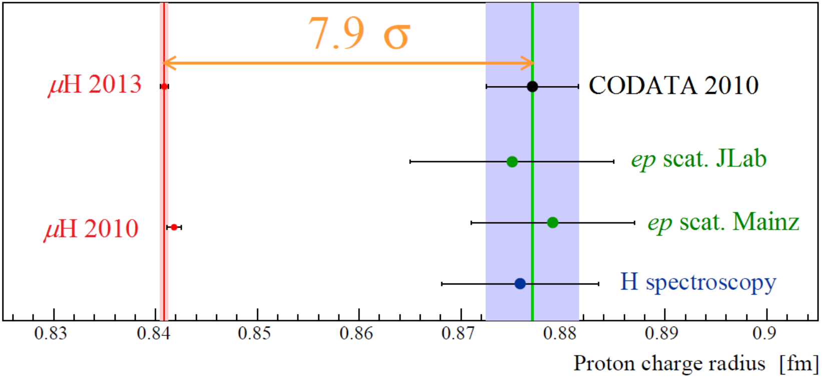
The Hebrew University of Jerusalem

Outline

- **Introduction**
  - The proton radius puzzle
  - Lamb shift, charge radius & nuclear structure
- **Calculation details**
- **Results**
  - μD
  - $\mu\,^4He^+$
  - $\mu\,^3He^+$
  - $\mu\,^3H$
- **Uncertainty estimates**
- **Summary**
- **Outlook**

How big is the proton?

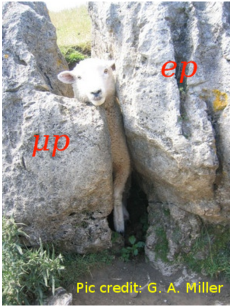


R. Pohl @ CREMA

Origins of the discrepancy?

What is the source of the discrepancy?

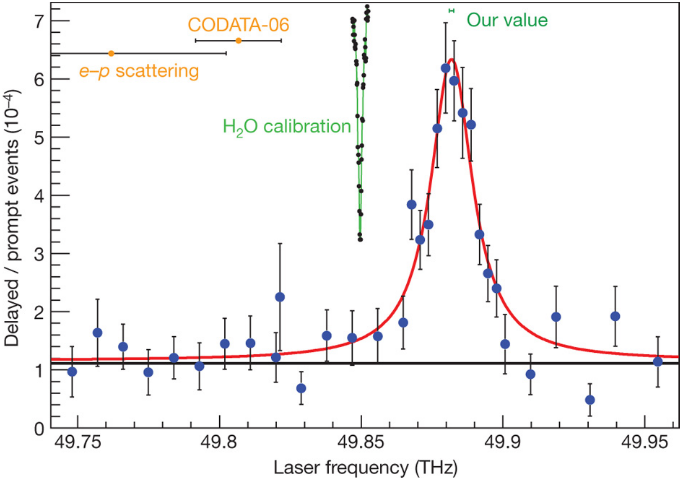
- μH experiment / theory?
- eH spectroscopy?
- ep scattering?
- overlooked/underestimated effects?
  (e.g., exotic hadron structure)



- beyond standard model?
  (extra dimensions, quantum gravity, new force-carriers, ...)

Origins of the discrepancy?

Problem with μH experiment / theory?



Origins of the discrepancy?

Problem with μH experiment / theory? — probably not

$$\widetilde{L}^{\text{theo.}}_{\mu p}(r_p^{\text{CODATA}}) - \widetilde{L}^{\text{exp.}}_{\mu p} = \begin{cases} 75\ \text{GHz} \\ 0.31\ \text{meV} \\ 0.15\ \% \end{cases}$$

μp theory wrong?

Discrepancy = 0.31 meV
Theory uncert. = 0.0025 meV

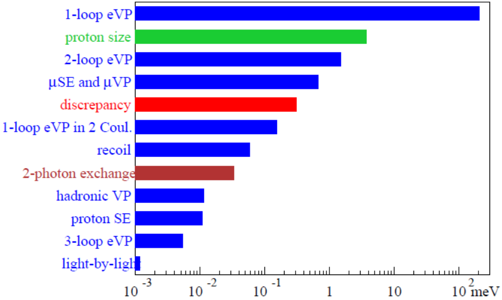
$\Longrightarrow$ $120\delta(\text{theory})$ deviation

$$\Delta E = 206.0668(25) - 5.2275(10)\, r_p^2 \ \text{[meV]}$$

double-checked by many groups

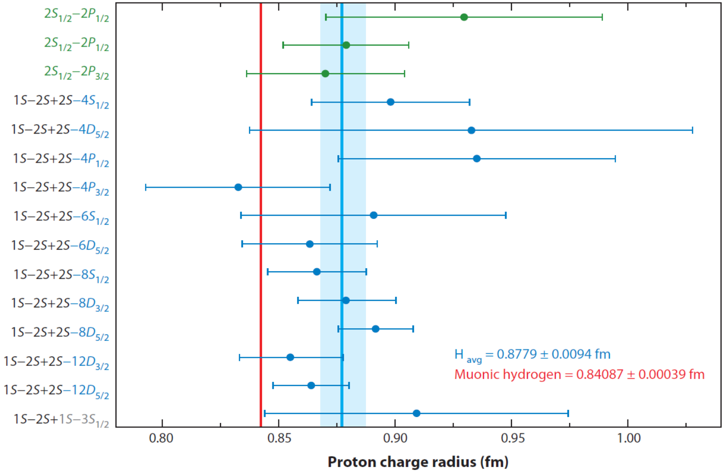
5^{th} largest term!

Some contributions to the μp Lamb shift



Theory summary:
A. Antognini, RP *et al.*
Annals of Physics 331, 127 (2013)

Problem with eH spectroscopy?



Pohl *et al.*, **Annu. Rev. Nucl. Part. Sci. (2013)**

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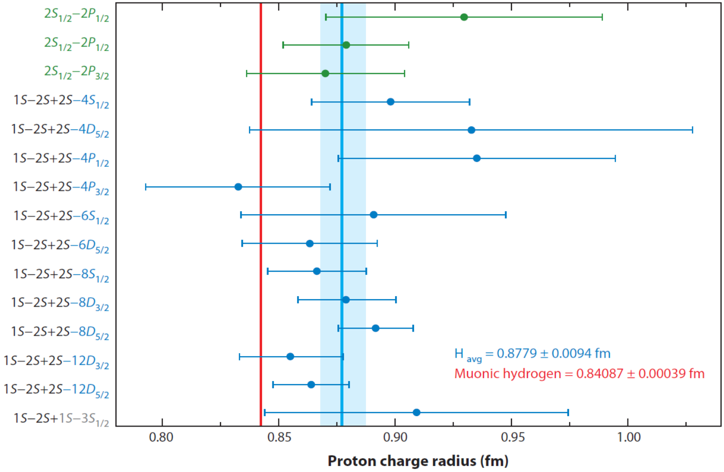

Pohl *et al.*, **Annu. Rev. Nucl. Part. Sci. (2013)**

Origins of the discrepancy?

Problem(s) with ep scattering?

$$G_E^p(Q^2) = 1 - \frac{1}{6}\boldsymbol{r_p^2}Q^2 + \ldots$$

$$\boldsymbol{r_p^2} \equiv -6\frac{dG_E^p(Q^2)}{dQ^2}\bigg|_{Q^2=0}$$

- Problem with measurements?
  - Q^2 not small enough?
- Problem with fits?
  - extrapolation to $Q^2 = 0$ depends on fit
  - various groups obtain contradicting results

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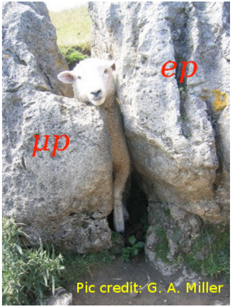
  Birse & McGovern, **EPJA '12** *vs.* Miller, **PLB '13**
  Hill & Paz, **PRD '10**; **PRL '11** & Jentschura **PRA '13**



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  - new force carriers, address also $(g-2)_\mu$ puzzle
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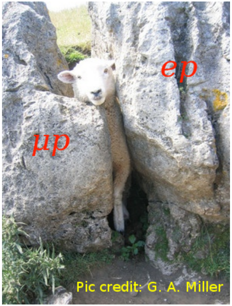

- beyond standard model? — possible (very constrained)
  - new force carriers, address also $(g-2)_\mu$ puzzle
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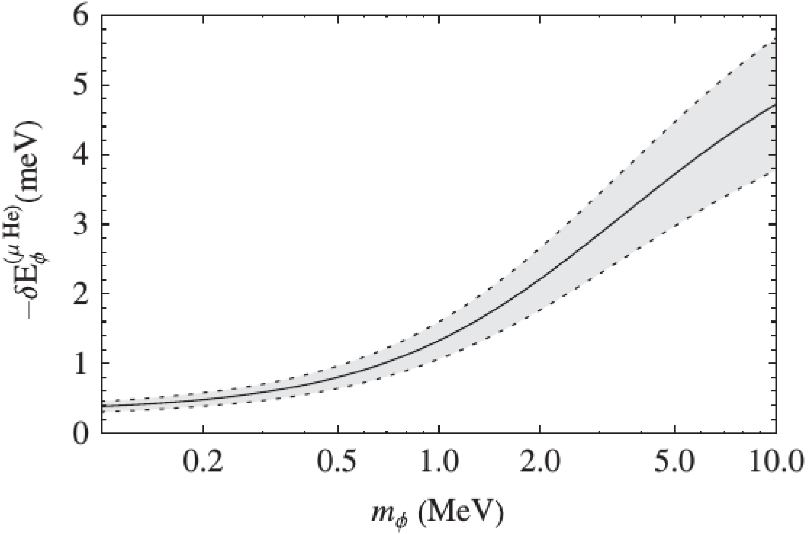
Tucker-Smith & Yavin's prediction for $\mu\,^4He^+$



Origins of the discrepancy? - more data!

New scattering experiments to shed light on the puzzle

JLab (windowless target) & MAMI (initial state radiation)

ep scattering for $Q^2 \geqslant 2 \times 10^{-4}$ GeV^2 (in progress)

MUSE collaboration at PSI

$\mu^{\pm}p$ (& $e^{\pm}p$) scattering experiment (in development)

MAMI electron-deuteron experiment

$e^{-}d$ scattering (data taken)

JLab ^{3}He & ^{3}H form factors

extract ^{3}He–^{3}H charge radii difference (planned)

Origins of the discrepancy? - more data!

New H-spectroscopy experiments to shed light on the puzzle

York Univ. (Toronto)

- 2S-2P "Ordinary" Lamb shift
- aim at 0.6% accuracy for r_p

MPQ (Garching)

- 2S-4P "Ordinary" transitions (+ 1S-2S)
- aim at 2% accuracy for r_p

NPL (UK)

- 2S-nS,D "Ordinary" transitions (+ 1S-2S)

LKB (Paris) & MPQ (Garching)

- 1S-3S "Ordinary" transitions (+ 1S-2S)
- two different methods, aim at 1% accuracy for r_p

NIST & ETHZ

- Rydberg const. — using circular Rydberg atoms & positronium

Origins of the discrepancy? - more data!

New μ-spectroscopy experiments to shed light on the puzzle

- **CREMA collaboration at PSI**
  - Lamb shift (2S-2P) in μD (finishing)
  - Lamb shift in $\mu\,^4$He$^+$ and $\mu\,^3$He$^+$ (both measured in 2014)
  - Lamb shift in $\mu\,^3$H, $\mu\,^6$He$^+$, and $Z = 3, 4, 5$ (wanted since '85)

    $\Longrightarrow$ Extract charge radii with high precision

    $\Longrightarrow$ proton puzzle, QED tests, He isotope shift, nuclear *ab initio*, ...

  - Hyperfine splitting (HFS) in μH & $\mu\,^3$He$^+$ (approved)

    $\Longrightarrow$ Extract magnetic radii

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  - HFS in μH (planned)

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high-precision measurements $\Longleftrightarrow$ accurate theoretical inputs

Lamb shift, charge radius & nuclear TPE

Extract $R_c \equiv \sqrt{\langle r^2 \rangle}$ from Lamb shift measurement

$$\Delta E_{2S-2P} = \delta_{QED} + \delta_{size}\left(R_c\right) + \delta_{\mathrm{Zem}} + \delta_{\mathrm{pol}}$$

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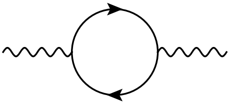
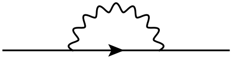
$$\Delta E_{2S-2P} = \delta_{QED} + \delta_{size}(R_c) + \delta_{\text{Zem}} + \delta_{\text{pol}}$$

- QED corrections:
  - vacuum polarization
  - lepton self energy
  - relativistic recoil effects





- Theory of μ-p, D, ^{3,4}He$^+$ reexamined

  Martynenko *et al.* **'07**, Borie **'12**, Krutov *et al.* **'15**

  Karshenboim *et al.* **'15**, Krauth *et al.* **'15** ...

Lamb shift, charge radius & nuclear TPE

Extract $R_c \equiv \sqrt{\langle r^2 \rangle}$ from Lamb shift measurement

$$\Delta E_{2S-2P} = \delta_{QED} + \delta_{size}\left(R_c\right) + \delta_{\text{Zem}} + \delta_{\text{pol}}$$

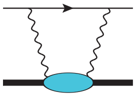
- Nuclear finite-size corrections (elastic):
  - leading term: $\delta_{size} = \frac{m_r^3}{12}\,(Z\alpha)^4 \times R_c^2$
  - Zemach/Friar term: $\delta_{\text{Zem}} = -\frac{m_r^4}{24}\,(Z\alpha)^5 \times \langle r^3 \rangle_{(2)} \quad \propto R_c^3$
    - can be calculated from g.s. charge distribution, Friar **'79**, Borie **'12('14)**, Krutov *et al.* **'15**
    - extracted from experimental form factors, Sick **'14**
    - or avoided due to cancellations with δ_{pol} Pachucki '11 & Friar '13 (μD)

Lamb shift, charge radius & nuclear TPE

Extract $R_c \equiv \sqrt{\langle r^2 \rangle}$ from Lamb shift measurement

$$\Delta E_{2S-2P} = \delta_{QED} + \delta_{size}(R_c) + \delta_{\mathrm{Zem}} + \delta_{\mathrm{pol}}$$

- Nuclear polarization corrections (inelastic):
  - least well-known
  - related to nuclear response functions:
    $S_O(\omega) = \sum\!\!\!\!\!\!\int_f |\langle\psi_f|\hat{O}|\psi_0\rangle|^2 \delta(E_f - E_0 - \omega)$
  - can be calculated (continuum few-body problem)
  - or extracted from data (very imprecise)
  - sometimes rewritten as:
    $\boldsymbol{\delta_{\mathrm{TPE}} \equiv \delta_{\mathrm{Zem}} + \delta_{\mathrm{pol}}}$



Case study — μD

$$\Delta E^{\rm LS}_{\rm QED} = 228.77356(75)\,{\rm meV}$$
$$\Delta E^{\rm LS}_{\rm rad.-dep.} = -6.11025(28)\,r_{\rm d}^2\,{\rm meV/fm}^2 + 0.00300(60)\,{\rm meV}$$
$$\Delta E^{\rm LS}_{\rm TPE} = 1.70910(2000)\,{\rm meV}$$

Krauth *et al.*, **Ann. Phys. (Mar. 2016)**

Previous calculations of $S_O(\omega)$ & $\delta_{\rm pol}$

- **μD (^{2}H)**
  - Modern forces: AV14 - Leidemann & Rosenfelder **'95**
  - State-of-the-art: AV18 - Pachucki **'11**
    - underestimates nuclear physics uncertainty
    - includes nucleon polarizability $\delta^{p}_{\rm pol}$ (partial)
  - $\not\pi$EFT: zero-range expansion - Friar **'13**
    - (under?) estimated uncertainty 1–2%
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- **Status of $\delta_{\rm pol}$ in light muonic atoms**
  - **experimental input** for S_O is unsatisfactory
  - need to calculate $\delta_{\rm pol}$ using **modern potentials and *ab-initio* methods**

Our work: $2 < A$

We have performed the first *ab-initio* calculation of δ_{pol} and δ_{Zem}

we use state-of-the-art forces

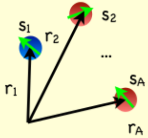
- **AV18+UIX**
- χ**EFT**

$\Longrightarrow$ estimate nuclear physics uncertainty

we employ established Few-body methods

- **EIHH**: Effective interaction Hyperspherical Harmonics (bound method)
- **LIT**: Lorentz Integral Transform (continuum method)
- **LSR**: A new method, based on the Lanczos algorithm
  NND *et al.*, **Phys. Rev. C (2016)**

Nuclear potentials: two approaches



$$H_N|\psi_i\rangle = E_i|\psi_i\rangle$$

$$H_N = T + V_{NN} + V_{3N} + \dots$$

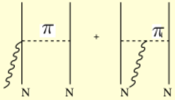
High precision two-nucleon potentials:
well constraint on NN phase shifts

Three nucleon forces:
less known, constraint on A>2 observables

Traditional Nuclear Physics
AV18+UIX, ..., J_2

Effective Field Theory
N^2LO, N^3LO ...

$$J^\mu = J^\mu_N + J^\mu_{NN} + \dots$$



two-body currents (or MEC)

subnuclear d.o.f.

J^μ consistent with V

$$\nabla \cdot J = -i[V, \rho]$$

$$S(\omega) \propto |\langle\psi_f|J^\mu|\psi_0\rangle|^2$$

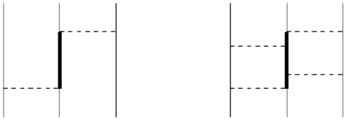
Exact Initial state &
Final state in the continuum at
different energies and for different A

Nuclear potentials: Traditional (phen.)

- **Argonne $v18$** fitted to
  - 1787 pp & 2514 np observables for $E_{lab} \leq 350\,\mathrm{MeV}$ with $\chi^2/\mathrm{datum} = 1.1$
  - nn scattering length & **D** binding energy

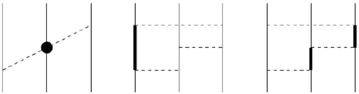
- **Urbana IX**

$$V_{ijk} = V_{ijk}^{2\pi P} + V_{ijk}^{R}$$



- **Illinois**

$$+V_{ijk}^{2\pi S} + V_{ijk}^{3\pi \Delta R}$$



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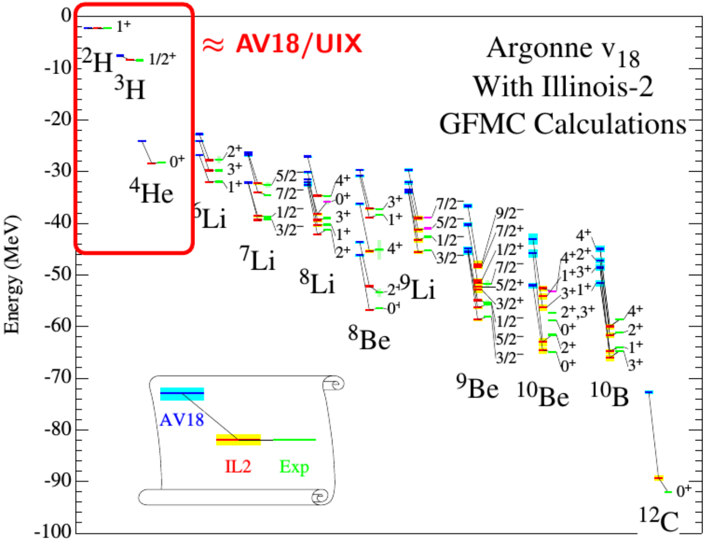
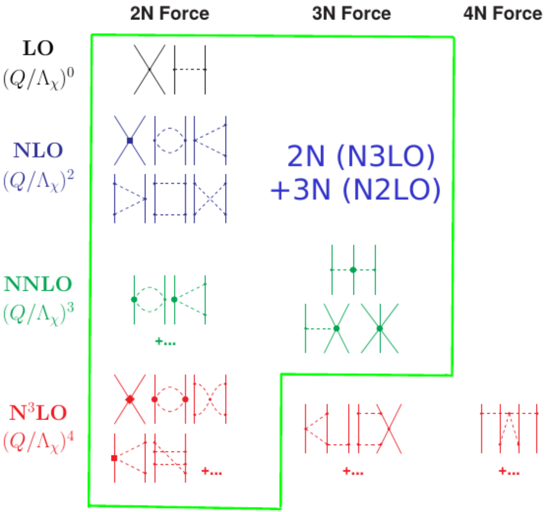


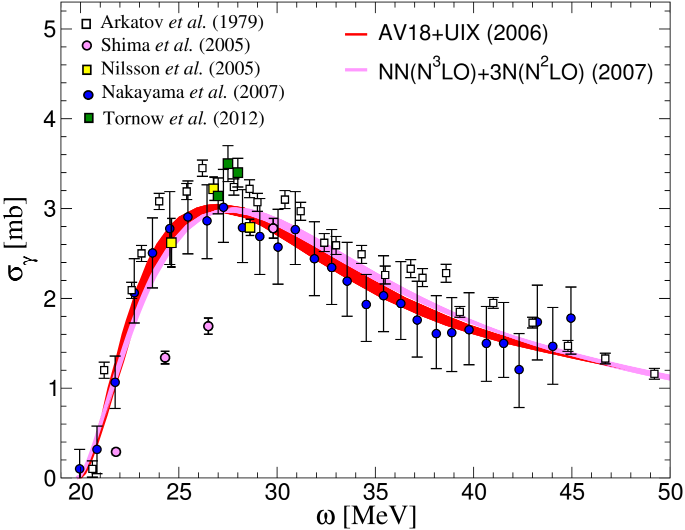
Nuclear potentials: Chiral-EFT

- **effective theory** of low-energy QCD
- **nuclear forces** are built in systematic expansions of Q/Λ
- **coupling constants** fitted to nuclear data



Nuclear potentials: ^{4}He Photoabsorption

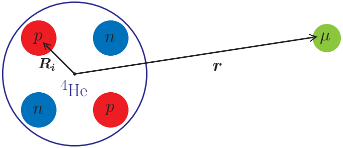
electric dipole photoabsorption cross section $\sigma_\gamma(\omega) = 4\pi^2 \alpha \omega S_{D1}(\omega)$



- Hamiltonian for muonic atoms

$$H = H_{nucl} + H_\mu + \Delta H$$

$$H_\mu = \frac{p^2}{2m_r} - \frac{Z\alpha}{r}$$



- Corrections to the point Coulomb from protons

$$\Delta H = \alpha \sum_i^Z \Delta V(\boldsymbol{r}, \boldsymbol{R}_i) \equiv \alpha \sum_i^Z \left(\frac{1}{r} - \frac{1}{|\boldsymbol{r} - \boldsymbol{R}_i|} \right)$$

- Evaluate inelastic effects of ΔH on muonic spectrum in 2^{nd}-order perturbation theory

$$\delta_{\text{pol}} = \sum_{N \neq N_0, \mu} \langle N_0 \mu_0 | \Delta H | N \mu \rangle \frac{1}{E_{N_0} - E_N + \epsilon_{\mu_0} - \epsilon_\mu} \langle N \mu | \Delta H | \mu_0 N_0 \rangle$$

$|\mu_0\rangle$: muon wave function for $2S/2P$ state

Nuclear polarization: contributions

Systematic contributions to nuclear polarization

δ_{NR} **Non-Relativistic** limit

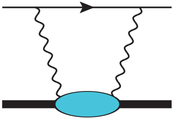
$\delta_L + \delta_T$ **L**ongitudinal and **T**ransverse **relativistic** corrections

δ_C **Coulomb** distortions

δ_{NS} Corrections from **finite Nucleon Size**

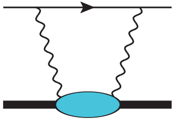
Non-relativistic limit

- Neglect Coulomb interactions in the intermediate state

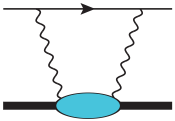


Non-relativistic limit

- Neglect Coulomb interactions in the intermediate state
- Expand muon matrix element in powers of $\eta \equiv \sqrt{2m_r\omega}|\boldsymbol{R} - \boldsymbol{R}'|$



Non-relativistic limit



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  - uncertainty principal $|\boldsymbol{R} - \boldsymbol{R}'| \sim 1/\sqrt{2m_N\omega}$
  - $\eta \sim \sqrt{\frac{m_r}{m_N}} \approx 0.3$

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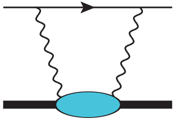

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  - $\eta \sim \sqrt{\frac{m_r}{m_N}} \approx 0.3$

$$P \simeq \frac{m_r^3(Z\alpha)^5}{12}\sqrt{\frac{2m_r}{\omega}}\left[|\boldsymbol{R}-\boldsymbol{R}'|^2 - \frac{\sqrt{2m_r\omega}}{4}|\boldsymbol{R}-\boldsymbol{R}'|^3 + \frac{m_r\omega}{10}|\boldsymbol{R}-\boldsymbol{R}'|^4\right]$$

$$\delta_{NR} = \delta_{NR}^{(0)} + \delta_{NR}^{(1)} + \delta_{NR}^{(2)} = \eta^2 \quad + \quad \eta^3 \quad + \quad \eta^4$$

$\delta_{NR} = \boldsymbol{\delta_{NR}^{(0)}} + \delta_{NR}^{(1)} + \delta_{NR}^{(2)}$

- $\delta_{NR}^{(0)} \propto \eta^2$

$$\delta_{D1}^{(0)} = -\frac{2\pi m_r^3}{9}(Z\alpha)^5 \int_{\omega_{\text{th}}}^{\infty} d\omega \sqrt{\frac{2m_r}{\omega}} S_{D_1}(\omega)$$

- $S_{D_1}(\omega) \Longrightarrow$ electric dipole response function [$\hat{D}_1 = R Y_1(\hat{R})$]
- $\delta_{D1}^{(0)}$ is the dominant contribution to δ_{pol}

NR limit at LO: $\delta_{NR}^{(0)}$

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- $\delta_{D1}^{(0)}$ is the dominant contribution to $\delta_{\rm pol}$
- $\Longrightarrow$ Rel. and Coulomb corrections added at this order

NR limit at NLO: $\delta_{NR}^{(1)}$

$$\delta_{NR} = \delta_{NR}^{(0)} + \boldsymbol{\delta_{NR}^{(1)}} + \delta_{NR}^{(2)}$$

- $\delta_{NR}^{(1)} \propto \eta^3$

$$\boxed{\delta_{NR}^{(1)} = \delta_{R3pp}^{(1)} + \delta_{Z3}^{(1)}}$$

$$\delta_{R3pp}^{(1)} = -\frac{m_r^4}{24}(Z\alpha)^5 \iint d\boldsymbol{R} d\boldsymbol{R}' |\boldsymbol{R} - \boldsymbol{R}'|^3 \langle N_0 | \hat{\rho}^\dagger(\boldsymbol{R}) \hat{\rho}(\boldsymbol{R}') | N_0 \rangle$$

$$\delta_{Z3}^{(1)} = \frac{m_r^4}{24}(Z\alpha)^5 \iint d\boldsymbol{R} d\boldsymbol{R}' |\boldsymbol{R} - \boldsymbol{R}'|^3 \rho_0(\boldsymbol{R}) \rho_0(\boldsymbol{R}')$$

- $\delta_{R3pp}^{(1)} \Rightarrow$ 3rd-order proton-proton correlation
- $\delta_{Z3}^{(1)} \Longrightarrow$ 3rd Zemach moment

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- $\delta^{(1)}_{Z3} \Longrightarrow$ 3rd Zemach moment
  cancels *elastic* Zemach moment of finite-size corrections
  c.f. Pachucki '11 & Friar '13 (μD)

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  c.f. Pachucki '11 & Friar '13 (μD) $\Longrightarrow \boldsymbol{\delta_{\mathrm{TPE}}} \equiv |\delta_{\mathrm{Zem}} + \delta_{\mathrm{pol}}|$

NR limit at N^{2}LO: $\delta_{NR}^{(2)}$

$\delta_{NR} = \delta_{NR}^{(0)} + \delta_{NR}^{(1)} + \boldsymbol{\delta_{NR}^{(2)}}$

- $\delta_{NR}^{(2)} \propto \eta^4$

$$\delta_{NR}^{(2)} = \frac{m_r^5}{18}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega \sqrt{\frac{\omega}{2m_r}} \left[S_{R^2}(\omega) + \frac{16\pi}{25} S_Q(\omega) + \frac{16\pi}{5} \mathcal{S}_{D_1 D_3}(\omega) \right]$$

- $S_{R^2}(\omega) \Longrightarrow$ monopole response function
- $S_Q(\omega) \Longrightarrow$ quadrupole response function
- $S_{D_1 D_3}(\omega) \Longrightarrow$ interference between D_1 and D_3 $\quad [\ \hat{D}_3 = R^3 Y_1(\hat{R})\ ]$

Test: $\delta^{(0)}$ in μD

- **Test Run: electric-dipole polarization effects in μD**
  - previous $\delta_{\rm pol}$ in $\boldsymbol{\mu}$D (AV18): Pachucki '11
  - we calculate $\delta^{(0)}$ from dipole response of D (AV18)
    [$S_{D_1}(\omega)$ from Bampa, Leidemann & Arenhövel '11]

$\delta^{(0)}$ [meV]		Pachucki '11	Our work
non-rel dipole	$\delta^{(0)}_{D1}$	-1.910	-1.907
relativistic	$\delta^{(0)}_{L}$	0.035	0.029
	$\delta^{(0)}_{T}$	–	-0.012
Coulomb	$\delta^{(0)}_{C}$	0.261	0.259

  - The difference in $\delta^{(0)}_{L}$ is due to small energy expansion used in Pachucki '11

He: Calculating few-body response functions

- **Few-body bound-states:**
  EIHH — Effective Interaction Hyperspherical Harmonics
  - Rapidly converging, based on symmetrized hyperspherical functions

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  - Transforms calculation of response $S(\omega)$ into two-steps:
    1. Solving a Schroedinger-like (bound-state) equation
    2. Inverting an integral transform (with Lorentzian kernel)

Efros, Leidemann, & Orlandini, **Phys. Lett. B (1994)**
Efros, Leidemann, Orlandini & Barnea, **J. Phys. G (2007)**

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Efros, Leidemann, & Orlandini, **Phys. Lett. B (1994)**
Efros, Leidemann, Orlandini & Barnea, **J. Phys. G (2007)**

  - Uses Lanczos to diagonalize low-energy spectrum of H_{nucl}

Marchisio, Barnea, Leidemann, & Orlandini, **Few-body Sys. (2003)**

LSR: Lanczos sum rule method

- Nuclear polarization $\Longrightarrow$ energy-dependent sum rules of the response functions

$$\delta_{\text{pol}} \propto I_O = \int_0^\infty d\omega \, S_O(\omega) \, g(\omega)$$

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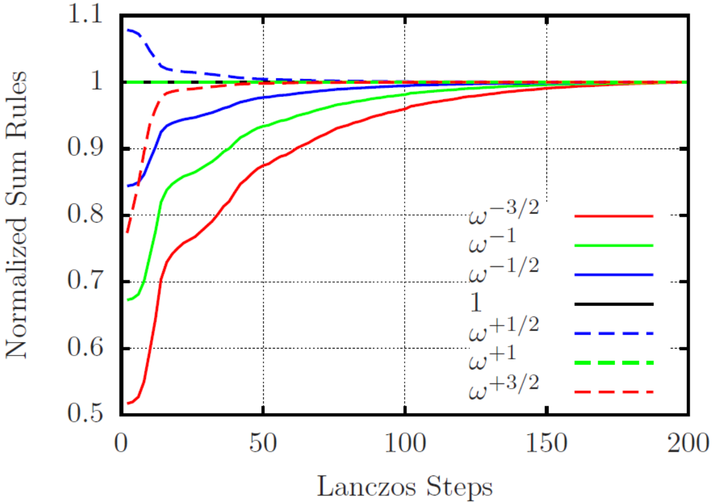
NND, Barnea, Ji, and Bacca, Phys. Rev. C **89**, 064317 (2014)

Example — ^{4}He dipole response sum-rules



(Model space size $M \sim 10^5$)

NND, Barnea, Ji, and Bacca, Phys. Rev. C **89**, 064317 (2014)

Nuclear polarization in $\mu\,^4\mathrm{He}^+$

[meV]		AV18+UIX	χEFT★
$\delta^{(0)}$	$\delta^{(0)}_{D1}$	-4.418	-4.701
	$\delta^{(0)}_{L}$	0.289	0.308
	$\delta^{(0)}_{T}$	-0.126	-0.134
	$\delta^{(0)}_{C}$	0.512	0.546

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$3N$: N^{2}LO (c_D=1, c_E=-0.029)

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	$\delta^{(1)}_{Z1}$	1.753	1.811
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[meV]	AV18+UIX	χEFT★
$\delta^{(0)}$	-3.743	-3.981
$\delta^{(1)}$	0.741	0.809
$\delta^{(2)}$	0.077	0.101
δ_{NS}	0.517	0.530
δ_{pol}	-2.408	-2.542

- Convergence from $\delta^{(0)}$ to $\delta^{(2)}$ from an expansion in $\eta \sim \sqrt{m_r/M_N} \approx 0.3$

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- Convergence from $\delta^{(0)}$ to $\delta^{(2)}$
  from an expansion in
  $\eta \sim \sqrt{m_r/M_N} \approx 0.3$
- δ_{pol} with AV18+UIX & χEFT differ:
  $\sim 5.5\%$ (0.134 meV)

★ NN: N^{3}LO-EM
$3N$: N^{2}LO (c_D=1, c_E=-0.029)

Nuclear physics uncertainty

^{4}He obesrvable		AV18+UIX	χEFT-EM	Difference
$\mu\,^4$He$^+$ nuclear polarization	$\delta_{\rm pol}$ [meV]	-2.408	-2.542	5.5%

Nuclear physics uncertainty

^{4}He obesrvable		AV18+UIX	χEFT-EM	Difference
binding energy	B_0 [MeV]	28.422	28.343	0.28%
point-proton nuclear radius	R_{pp} [fm]	1.432	1.475	3.0%
electric-dipole polarizability	α_E [fm^3]	0.0651	0.0694	6.4%
$\mu\,^4$He$^+$ nuclear polarization	$\delta_{\rm pol}$ [meV]	-2.408	-2.542	5.5%

- B_0, R_{pp} & α_E in good agreement with previous calculations
  Kievsky *et al.* '08, Gazit *et al.* '06 & Stetcu *et al.* '09

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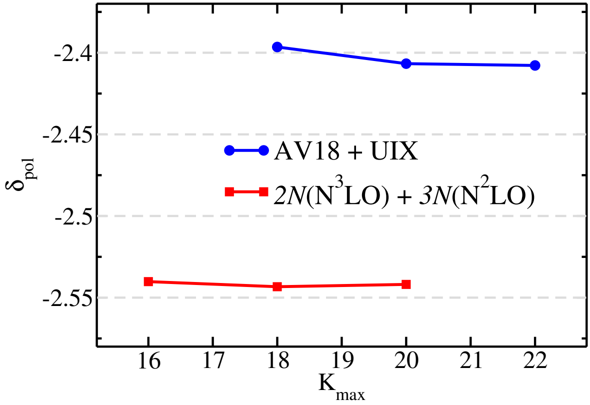
- B_0, R_{pp} & α_E in good agreement with previous calculations
  Kievsky *et al.* '08, Gazit *et al.* '06 & Stetcu *et al.* '09
- systematic uncertainty in $\delta_{\rm pol}$ from nuclear physics:
  $\frac{5.5\%}{\sqrt{2}} \implies \pm 4\%\ (1\sigma)$

Convergence with model space size

Compare $\delta_{\text{pol}}^{(K_{max})}$ with $\delta_{\text{pol}}^{(K_{max}-4)}$

- AV18+UIX $\sim 0.4\%$
- χEFT-EM $\sim 0.2\%$



Error budget — $\mu\,^4\text{He}^+$

- **Nuclear physics**

  4% from two potentials

- **Numerical accuracy**

  0.4% from convergence

- **Additional corrections**
  - $(Z\alpha)^6$ terms (beyond 2nd-order perturbation theory)
  - Rel. & Coulomb corrections (other than dipole)
  - higher-order nucleon-size corrections

  $\Rightarrow$ $\sim 4\%$ estimated from additional corrections

- **Final result (quadratic sum)**

our prediction:	$\delta_{\text{pol}} = -2.47$ meV $\pm 6\%$
previous estimates:	$\delta_{\text{pol}} = -3.1$ meV $\pm 20\%$
experimental needs:	δ_{pol} uncertainty $\sim 5\%$

Reflections on $\mu\,^4\text{He}^+$

- The accuracy of δ_{pol} in $\mu\,^4\text{He}^+$ is limited by the nuclear physics (AV18+UIX vs. χEFT-EM)
- To study this we can further vary the nuclear potentials
  - Use χEFT at different orders to track the convergence
  - At each order vary the cutoff to estimate the theoretical error (χEFT-EGM: Epelbaum, Glöckle, & Meißner, **Nucl. Phys. A '05**)

Reflections on μD

- Pachucki only used AV18
  - $\Rightarrow$ No nuclear physics uncertainty
  - $\Rightarrow$ We can add χEFT-EM & χEFT-EGM
- 2-body problem
  - $\Rightarrow$ only NN interaction
  - $\Rightarrow$ simple numerics
- Other issues
  - Pachucki did not include nucleon-size corrections
  - Pachucki did not treat nucleon-polarization correctly
  - We already reproduced Pachucki's leading term as a check for $\mu\,^4$He$^+$
  - There is also a Magnetic contribution
  - Pachucki only calculated $\delta_{\rm TPE} \equiv |\delta_{\rm Zem} + \delta_{\rm pol}|$

Nuclear polarization in μD

	Pachucki '11 (AV18)	our work (AV18)
$\delta^{(0)}_{D1}$	-1.910	-1.907
$\delta^{(0)}_{L}$	0.035	0.029
$\delta^{(0)}_{T}$	—	-0.012
$\delta^{(0)}_{C}$	0.261	0.262
$\delta^{(2)}_{R2}$	0.045	0.042
$\delta^{(2)}_{Q}$	0.066	0.061
$\delta^{(2)}_{D1D3}$	-0.151	-0.139
$\delta^{(1)}_{NS}$	—	0.017
$\delta^{(2)}_{NS}$	—	-0.015
δ_{M}	0.016	0.008
$\delta_{\rm TPE} \equiv \lvert\delta_{\rm pol} + \delta_{\rm Zem}\rvert$	**1.638**	**1.656**

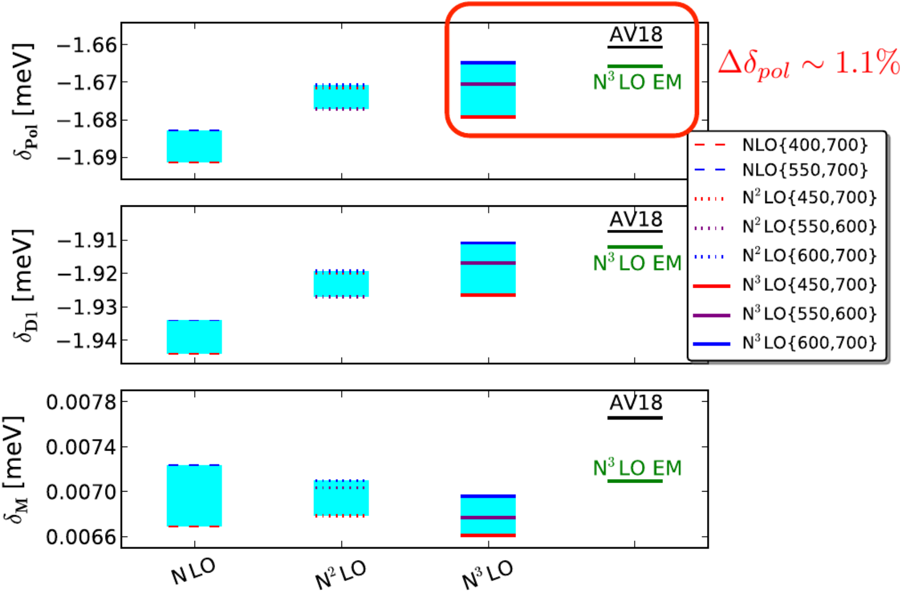
- compare: $\delta^{(0)}_{L}$ & $\delta^{(0)}_{T}$; $\delta^{(2)}$; δ_{M} ; δ_{NS}

Nuclear polarization in μD

	Pachucki '11 (AV18)	our work (AV18)	our work N^3LO-EM	our work N^3LO-EGM
$\delta^{(0)}_{D1}$	-1.910	-1.907	-1.912	(-1.911,-1.926)
$\delta^{(0)}_{L}$	0.035	0.029	0.029	(0.029, 0.030)
$\delta^{(0)}_{T}$	—	-0.012	-0.012	-0.013
$\delta^{(0)}_{C}$	0.261	0.262	0.262	(0.262, 0.264)
$\delta^{(2)}_{R2}$	0.045	0.042	0.041	0.041
$\delta^{(2)}_{Q}$	0.066	0.061	0.061	0.061
$\delta^{(2)}_{D1D3}$	-0.151	-0.139	-0.139	(-0.139,-0.140)
$\delta^{(1)}_{NS}$	—	0.017	0.017	0.017
$\delta^{(2)}_{NS}$	—	-0.015	-0.015	-0.015
δ_{M}	0.016	0.008	0.007	0.007
$\delta_{\rm TPE} \equiv \lvert\delta_{\rm pol} + \delta_{\rm Zem}\rvert$	**1.638**	**1.656**	**1.661**	**(1.660,1.674)**

- compare: $\delta^{(0)}_{L}$ & $\delta^{(0)}_{T}$; $\delta^{(2)}$; δ_{M} ; δ_{NS}

Improved nuclear uncertainty in μD



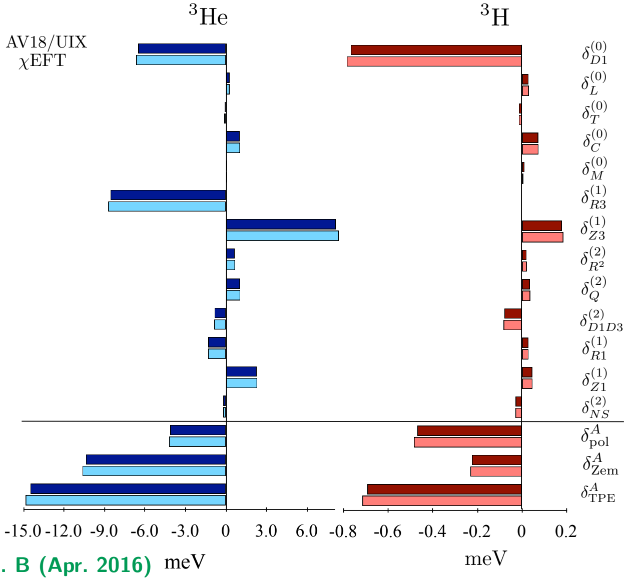
Error budget — μD

- **Nuclear physics**
  $\sim 1.1\%$ from a range of potentials
- **Numerical accuracy**
  (negligible)
- **Additional corrections**
  0.95% (as estimated by Pachucki)
- **Final result (quadratic sum)**
  our prediction: $\delta_{\rm TPE} \equiv |\delta_{\rm pol} + \delta_{\rm Zem}| = 1.66$ meV $\pm 1.5\%$
  includes nuclear error & nucleon-size corrections
  $\Rightarrow$ improved result and error estimate

μD theory: Krauth *et al.*, Ann. Phys. (Mar. 2016)

Item	Contribution	Pachucki [55] AV18		Friar [60] ZRA		Hernandez *et al.* [58] AV18	N^3LO †		Pach.& Wienczek [65] AV18	
	Source	1		2		3	4		5	
p1	Dipole	1.910	$\delta_0 E$	1.925	Leading C1	1.907	1.926	$\delta_{D1}^{(0)}$	1.910	$\delta_0 E$
p2	Rel. corr. to p1, longitudinal part	−0.035	$\delta_R E$	−0.037	Subleading C1	−0.029	−0.030	$\delta_L^{(0)}$	−0.026	$\delta_R E$
p3	Rel. corr. to p1, transverse part					0.012	0.013	$\delta_T^{(0)}$		
p4	Rel. corr. to p1, higher order								0.004	$\delta_{HO} E$
sum	Total rel. corr., p2+p3+p4	−0.035		−0.037		−0.017	−0.017		−0.022	
p5	Coulomb distortion, leading	−0.255	$\delta_{C1} E$						−0.255	$\delta_{C1} E$
p6	Coul. distortion, next order	−0.006	$\delta_{C2} E$						−0.006	$\delta_{C2} E$
sum	Total Coulomb distortion, p5+p6	−0.261				−0.262	−0.264	$\delta_C^{(0)}$	−0.261	
p7	El. monopole excitation	−0.045	$\delta_{Q0} E$	−0.042	C0	−0.042	−0.041	$\delta_{R2}^{(2)}$	−0.042	$\delta_{Q0} E$
p8	El. dipole excitation	0.151	$\delta_{Q1} E$	0.137	Retarded C1	0.139	0.140	$\delta_{D1D3}^{(2)}$	0.139	$\delta_{Q1} E$
p9	El. quadrupole excitation	−0.066	$\delta_{Q2} E$	−0.061	C2	−0.061	−0.061	$\delta_Q^{(2)}$	−0.061	$\delta_{Q2} E$
sum	Tot. nuclear excitation, p7+p8+p9	0.040		0.034	C0 + ret-C1 + C2	0.036	0.038		0.036	
p10	Magnetic	−0.008 [◇a]	$\delta_M E$	−0.011	M1	−0.008	−0.007	$\delta_M^{(0)}$	−0.008	$\delta_M E$
SUM_1	Total nuclear (corrected)	1.646		1.648 [b]		1.656	1.676		1.655	
p11	Finite nucleon size			0.021	Retarded C1 f.s.	0.020 [◇c]	0.020 [◇c]	$\delta_{NS}^{(2)}$	0.020	$\delta_{FS} E$
p12	n p charge correlation			−0.023	pn correl. f.s.	−0.017	−0.017	$\delta_{np}^{(1)}$	−0.018	$\delta_{FZ} E$
sum	p11+p12			−0.002		0.003	0.003		0.002	
p13	Proton elastic 3rd Zemach moment	} 0.043(3)	$\delta_P E$	0.030	$\langle r^3 \rangle_{(2)}^{pp}$				} 0.043(3)	$\delta_P E$
p14	Proton inelastic polarizab.					} 0.027(2)		δ_{pol}^N [64]		
p15	Neutron inelastic polarizab.								0.016(8)	$\delta_N E$
p16	Proton & neutron subtraction term									
sum	Nucleon TPE, p13+p14+p15+p16	0.043(3)		0.030		0.027(2)			0.059(9)	
SUM_2	Total nucleon contrib.	0.043(3)		0.028		0.030(2)			0.061(9)	
	Sum, published	1.680(16)		1.941(19)		1.690(20)			1.717(20)	
	Sum, corrected			1.697(19) [g]		1.714(20) [h]			1.707(20) [i]	

Hot from the press: $A = 3$



NND *et al.*, **Phys. Lett. B (Apr. 2016)**

Hot from the press: $\delta_{\rm pol}$ in $\mu\,^3\mathrm{He}^+$

[meV]	AV18+UIX	χEFT★
$\delta^{(0)}$	-5.346	-5.486
$\delta^{(1)}$	-0.438	-0.384
$\delta^{(2)}$	0.806	0.830
δ_{NS}	0.783	0.792
δ_{Mag}	0.081	0.047
$\delta_{\rm pol}$	-4.114	-4.201

- Convergence from $\delta^{(0)}$ to $\delta^{(2)}$?

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  c.f. $\mu\,^4$He$^+$:

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Hot from the press: $\delta_{\rm pol}$ in $\mu\,^3$H

[meV]	AV18+UIX	χEFT*
$\delta^{(0)}$	-0.679	-0.696
$\delta^{(1)}$	0.178	0.184
$\delta^{(2)}$	-0.024	-0.025
δ_{NS}	0.046	0.047
δ_{Mag}	0.010	0.006
$\delta_{\rm pol}$	-0.469	-0.483

- Convergence from $\delta^{(0)}$ to $\delta^{(2)}$?
- AV18+UIX vs. χEFT: $\sim 1.5\%$ (± 7.5 μeV)
- Probe $\delta^N_{\rm pol}$ of the nucleons ?

  $\delta^N_{\rm pol} \approx 34(16)$ μeV
- Precise triton radius

$\star$ NN: N^{3}LO-EM
$3N$: N^{2}LO (c_D=1, c_E=-0.029)

Uncertainty estimates

Error type	$\mu\,^3\mathrm{He}^+$ δ^A_{pol}	δ^A_{Zem}	δ^A_{TPE}	$\mu\,^3\mathrm{H}$ δ^A_{pol}	δ^A_{Zem}	δ^A_{TPE}
Numerical	0.4	0.1	0.1	0.1	0.0	0.1
Nuclear model	1.5	1.8	1.7	2.2	2.3	2.2
ISB	2.0	0.2	0.5	0.9	0.2	0.6
Nucleon size	1.6	1.5	0.6	0.6	1.3	0.0
Relativistic	0.6	-	1.5	1.4	-	0.3
Coulomb	1.2	-	0.3	0.3	-	0.2
Multipole expansion	2.0	-	0.6	2.0	-	1.4
Higher $Z\alpha$	1.5	-	0.4	0.7	-	0.5
Magnetic MEC	0.4	-	0.1	0.3	-	0.2
Total	4.1%	2.3%	2.5%	3.6%	2.7%	2.7%

Summary of results: $2 \leqslant A \leqslant 4$

System	Ref.	δ_{pol} err.	exptl. status
$\mu\,^2$H	Phys. Lett. B '14	1.3%	measured, unpublished
$\mu\,^4$He$^+$	Phys. Rev. Lett. '13	20% $\to$ 6%	measured, unpublished
$\mu\,^3$He$^+$	Phys. Lett. B '16	20% $\to$ 4%	measured, unpublished
$\mu\,^3$H	Phys. Lett. B '16	4%	measurable?

- δ_{Zem} agree with other values calculated or extracted from data
- δ_{pol} more accurate than previous estimates
- δ_{pol} err. comparable to the $\sim 5\%$ experimental needs
- will significantly improve the precision of R_c extracted from the measured Lamb shifts
- may help shed light on the proton radius puzzle

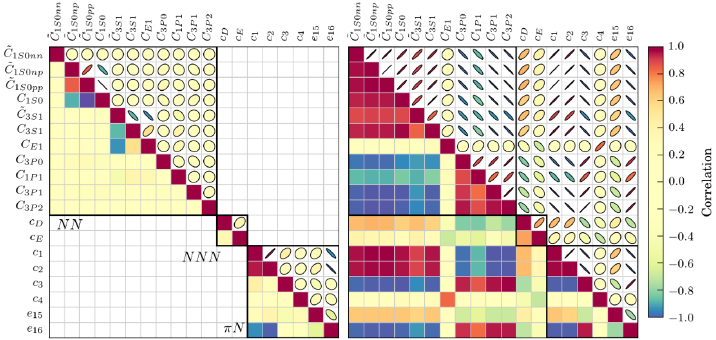
Work in progress

The work is not completed yet ...

Outlook

- Study higher-order terms (in progress)
- Quantify & reduce nuclear physics uncertainty (in progress)
  - **understand** why various nuclear Hamiltonians differ
  - further **explore** the phase-space of nuclear Hamiltonians
  - include higher-order or otherwise **improved nuclear forces**
  - include many-body currents
- Improve treatment of nucleon finite sizes (in progress)
- Investigate nuclear corrections in $\mu\,^6\mathrm{Li}^{+2}$, $\mu\,^6\mathrm{He}^{+}$, $\cdots$
- Investigate nuclear corrections in HFS of $\mu\,^3\mathrm{He}^{+}$

Quantitative nuclear physics uncertainty



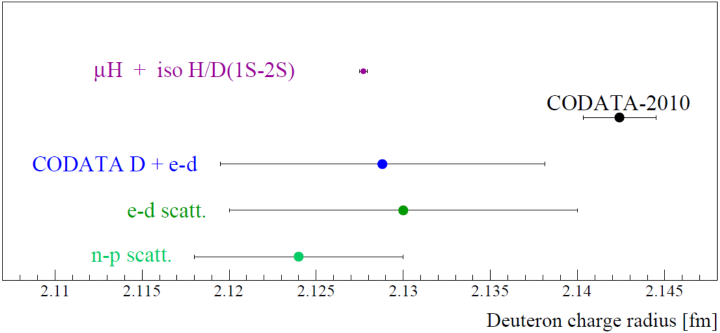
B. D. Carlsson et al., Phys. Rev. X (2016)

Update from CREMA: μD status 2015

H/D isotope shift: $r_d^2 - r_p^2 = 3.82007(65)$ fm^2 C.G. Parthey, RP *et al.*, PRL **104**, 233001 (2010)

CODATA 2010 r_d = 2.1424(21) fm

r_p= 0.84087(39) fm from μH gives r_d = 2.1277(2) fm



Courtesy of Randolf Pohl @ CREMA

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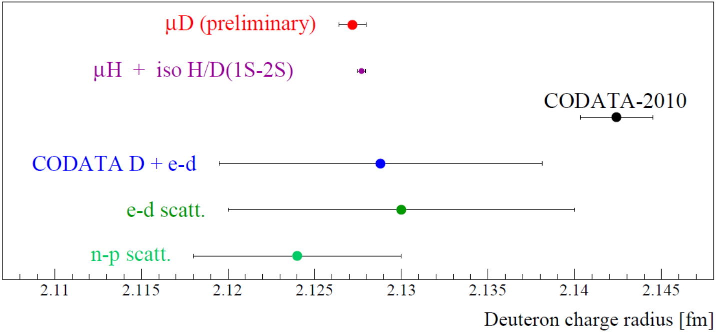
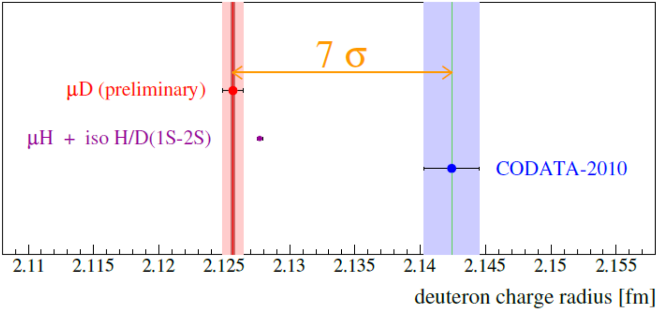
r_p= 0.84087(39) fm from μH gives	r_d = 2.1277(2) fm
Lamb shift in muonic DEUTERIUM	r_d = 2.1272(12) fm PRELIMINARY!



Courtesy of Randolf Pohl @ CREMA

Update from CREMA: μD status 2015

- Deuteron charge radius $r_{\mathrm{d}} = 2.12XX(8)$ fm
- Close to extraction from μH & isotope shift (1S-2S)
- Not in agreement with 2010 CODATA value



Courtesy of Randolf Pohl @ CREMA

The muonic Lamb shift is still puzzling !!!

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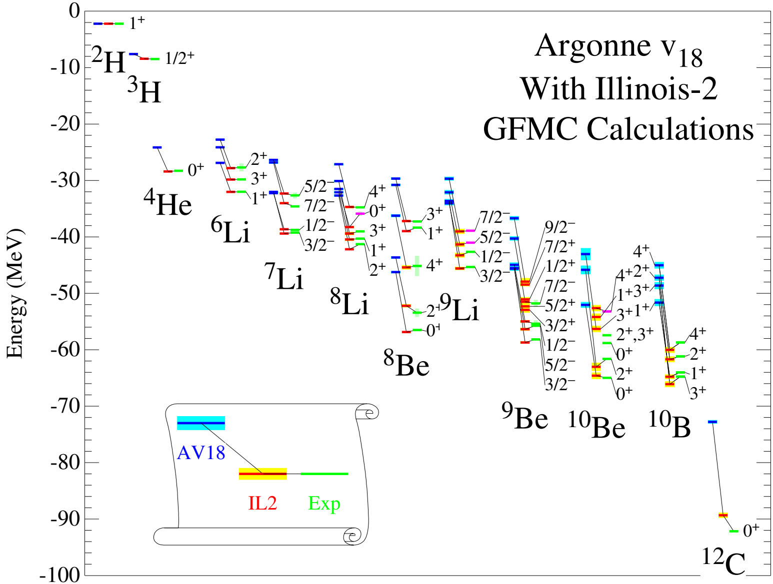
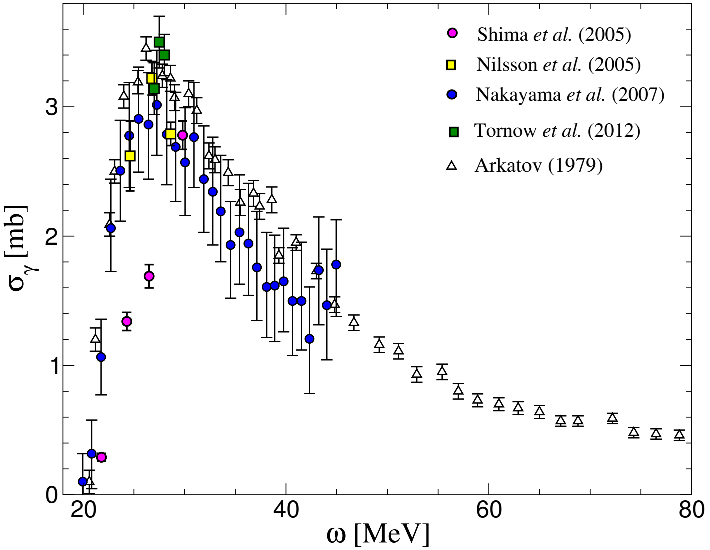
BACK UP

Phenomenological potentials



^{4}He photoabsorption cross sections

electric-dipole photoabsorption cross section $\sigma_\gamma(\omega) = 4\pi^2\alpha\omega S_{D1}(\omega)$



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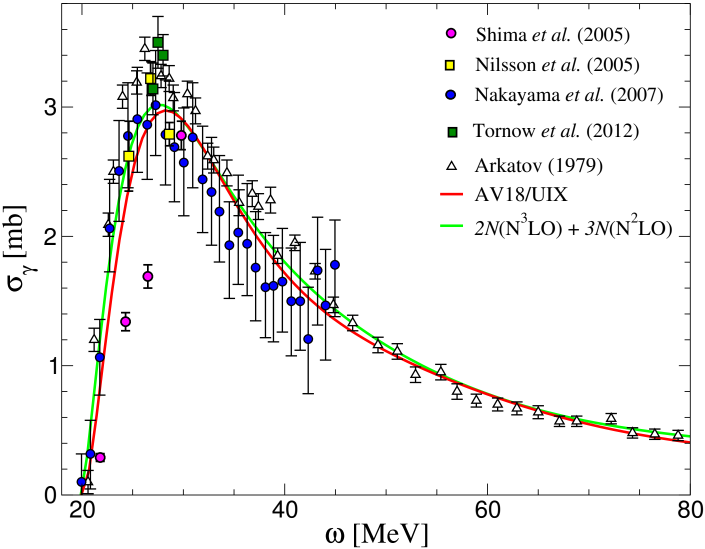

Non-Relativistic Approximation

- Coulomb interactions in the (virtual) intermediate state are neglected
- Results are expressed in an expansion of $\sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'|$

$\Delta E_{NR} = \Delta E_{NR}^{(2)} + \Delta E_{NR}^{(3)} + \Delta E_{NR}^{(4)}$

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1. $\sim R^2$ **term:**

- $\Delta E_{NR}^{(2)}$ **is the dominant polarizability contribution**

$$\Delta E_{NR}^{(2)} = -\frac{2\pi m_r^3}{9}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega \sqrt{\frac{2m_r}{\omega}} S^{D_1}(\omega)$$

- $S^{D_1}(\omega) = \frac{1}{2J_0+1} \sum_{N\neq N_0,J} |\langle N_0 J_0 || \hat{D}_1 || N J\rangle|^2 \delta(\omega - E_N + E_{N_0})$
- $\hat{D}_1 = \frac{1}{Z}\sum_i^Z R_i Y_1(\hat{R}_i)$

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2. $\sim R^3$ **term:**

$$\Delta E_{NR}^{(3)} = -\frac{m_r^4}{24}(Z\alpha)^5 \left[\iint d^3R d^3R' |\boldsymbol{R}-\boldsymbol{R}'|^3 \langle N_0|\hat{\rho}^\dagger(\boldsymbol{R})\hat{\rho}(\boldsymbol{R}')|N_0\rangle - \langle r^3\rangle_{(2)}\right]$$

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- 1st term: charge correlation function
  vanishes in point-nucleon limit

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- 2nd term: Zemach moment
  $\langle r^3\rangle_{(2)} = \iint d^3R d^3R' |\boldsymbol{R}-\boldsymbol{R}'|^3 \rho_0(\boldsymbol{R})\,\rho_0(\boldsymbol{R}')$
  $\rho_0(\boldsymbol{R}) = \langle N_0|\hat{\rho}(\boldsymbol{R})|N_0\rangle$

  cancels exactly the **Zemach term** in (elastic) finite-size corrections
  c.f. Pachucki PRL 2011 (μD)

3. $\sim R^4$ **term:**

- $\Delta E_{NR}^{(4)}$ **corresponds to higher-multipole corrections**

$$\Delta E_{NR}^{(4)} = \frac{m_r^5}{18}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega \sqrt{\frac{\omega}{2m_r}} \left[S^{Q_0}(\omega) + \frac{16\pi}{25} S^{Q_2}(\omega) + \frac{16\pi}{5} S^{D_{13}}(\omega) \right]$$

- $S^{R^2}(\omega) = \frac{1}{2J_0+1} \sum_{N \neq N_0, J} |\langle N_0 J_0 || \hat{R}^2 || N J \rangle|^2 \delta(\omega - E_N + E_{N_0})$

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  $$S^{D_{13}}(\omega) = \frac{1}{2J_0+1} \sum_{N \neq N_0, J} (-1)^{J_0 - J} \times \mathrm{Re}\left(\langle N_0 J_0 || \hat{D}_3 || N J \rangle \langle N J || \hat{D}_1 || N_0 J_0 \rangle \right) \delta(\omega - E_N + E_{N_0})$$

- $\hat{R}^2 = \frac{1}{Z} \sum_i^Z R_i^2$ $\qquad$ $\hat{D}_1 = \frac{1}{Z} \sum_i^Z R_i Y_1(\hat{R}_i)$

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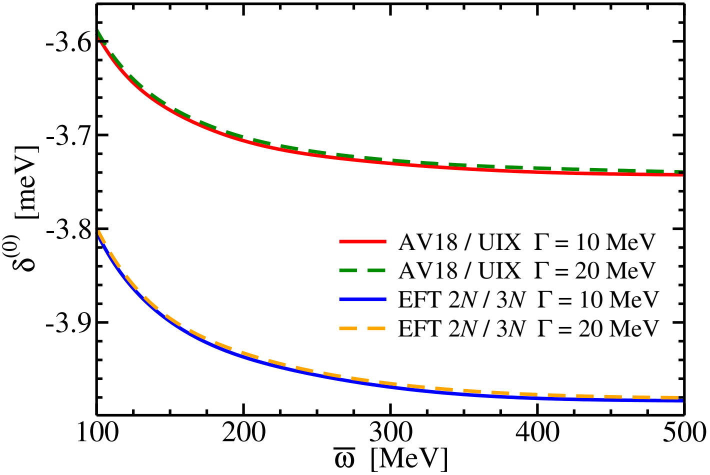
Convergence of $\bar{\omega}$



Convergence of Ab-initio calculations

$\delta^{(0)}$ convergence with the largest model space K_{max}